Name of College - [illegible] College, [illegible]
Subject - Mathematics
Topic - Problem on Indeterminate Form (contd.) 01
(Differential Calculus) [illegible]
Time - 10:15 to 11:00 AM
and 11:00 to 11:45 AM
Date - [illegible]
By - [illegible] Kumar

* Problem based on the form $0^0, \infty^0, 1^\infty$

1. Evaluate

$$\lim_{x \to 0} (\sin x)^{\tan x}$$

Solution :→ Let $y = \lim_{x \to 0} (\sin x)^{\tan x}$ (0^0)

$$\Rightarrow \log_e y = \lim_{x \to 0} \tan x \log \sin x \quad [0 \times \infty]$$

$$= \lim_{x \to 0} \frac{\log \sin x}{\cot x} \quad [\infty/\infty]$$

$$= \lim_{x \to 0} \frac{\frac{1}{\sin x} \cos x}{-\operatorname{cosec}^2 x} \quad [\text{By L Hospital's Rule}]$$

$$= \lim_{x \to 0} (-\sin x \cos x) = 0$$

$$\Rightarrow \log_e y = 0 \Rightarrow y = e^0 = 1$$

Problem 2 : Evaluate $\lim_{x \to 0} x^x$

[it is of the form 0^0]

Let $y = \lim_{x \to 0} x^x$

$$\Rightarrow \log y = \lim_{x \to 0} x \log x \quad [\text{Form } 0 \times \infty]$$

$$= \lim_{x \to 0} \frac{\log x}{1/x} \quad [\infty/\infty]$$

Applying L Hospital Rule

$$\log y = \lim_{x \to 0} \frac{1/x}{-1/x^2}$$

$$= \lim_{x \to 0} -x = 0$$

$$\therefore y = e^0 = 1$$

Evaluate $\lim_{x \to \pi/2} (\cos x)^{\cos x}$ [it is of the form 0^0]

Let $y = \lim_{x \to \pi/2} (\cos x)^{\cos x}$

$$\Rightarrow \log y = \lim_{x \to \pi/2} \cos x \log \cos x \quad \text{form } 0 \times \infty$$

$$= \lim_{x \to \pi/2} \frac{\log \cos x}{\sec x} \quad [\infty/\infty]$$

$$= \lim_{x \to \pi/2} \frac{\frac{1}{\cos x}(-\sin x)}{\sec x \tan x}$$

$$= \lim_{x \to \pi/2} -\frac{1}{\sec x}$$

$$= \lim_{x \to \pi/2} -\cos x = \cos \pi/2 = 0$$

$$\therefore y = e^0 = 1$$

Evaluate $\lim_{x \to \pi/2} (\sec x)^{\cot x}$ →

Solution → Let $y = \lim_{x \to \pi/2} (\sec x)^{\cot x}$

then $\log y = \lim_{x \to \pi/2} \cot x \log \sec x$

$= \lim_{x \to \pi/2} \dfrac{\log \sec x}{\tan x}$ $[\infty/\infty]$

$= \lim_{x \to \pi/2} \dfrac{\frac{1}{\sec x} \cdot \sec x \tan x}{\sec^2 x}$

$= \lim_{x \to \pi/2} \dfrac{\tan x}{\sec^2 x}$

$= \lim_{x \to \pi/2} \sin x \cos x$

$= 0$

Hence $\log y = 0$

$\Rightarrow y = e^0 = 1$

Find the limit of

$(\cot x)^{1/\log x}$ at $x = 0$

Solution :-> ~~log~~

$y = \lim_{x \to 0} (\cot x)^{1/\log x}$ $[\infty^0]$

$\Rightarrow \log y = \lim_{x \to 0} \dfrac{1}{\log x} \log \cot x$

$= \lim_{x \to 0} \dfrac{\log \cot x}{\log x}$ $\left[\dfrac{\infty}{\infty}\right]$

By using L-Hospital Rule.

We have $\log y = \dfrac{\lim_{x\to 0} \frac{1}{\cot x}(-\csc^2 x)}{1/x}$

$= \lim_{x\to 0} \dfrac{x}{\sin x \cos x} \quad \left[\frac{0}{0}\right]$

$= -\lim_{x\to 0} \dfrac{1}{\cos^2 x - \sin^2 x}$

$= -1$

Thus $\log y = -1$

$\Rightarrow y = e^{-1} = 1/e$

$\therefore$ Required limit $= 1/e$

Problem: Evaluate

~~Evalu~~ $\lim_{x\to 0} (\cos x)^{\cot^2 x}$

Let $y = \lim_{x\to 0} (\cos x)^{\cot^2 x}$

$\Rightarrow \log y = \lim_{x\to 0} \cot^2 x \log \cos x$

$= \lim_{x\to 0} \dfrac{\log \cos x}{\tan^2 x} \quad \left[\frac{0}{0}\right]$

Now using L-Hospital Rule.

$\Rightarrow \log y = \lim_{x\to 0} \dfrac{\frac{1}{\cos x}(-\sin x)}{2\tan x \cdot \sec^2 x}$

$= -\frac{1}{2} \lim_{x\to 0} \cos^2 x = -1/2$

$\therefore y = e^{-1/2} = \dfrac{1}{e^{1/2}} = \dfrac{1}{\sqrt{e}}$

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Applying L-Hospital Rule.

$$\log y = \lim_{x \to \pi/2} \frac{\frac{1}{\sin x}\cos x}{-\operatorname{cosec}^2 x}$$

$$= -\lim_{x \to \pi/2} \frac{\cot x}{\operatorname{cosec}^2 x}$$

$$= -\lim_{x \to \pi/2} \cos x \sin x$$

$$= -0 \times 1 = +0$$

$$\therefore y = e^0 = 1$$

Problem :→ Evaluate $\lim_{x \to 0} \left(\frac{\sin x}{x}\right)^{1/x^2}$ 1^∞

Solution → Let $y = \lim_{x \to 0} \left(\frac{\sin x}{x}\right)^{1/x^2}$

$$\Rightarrow \log y = \lim_{x \to 0} \frac{1}{x^2} \log \frac{\sin x}{x}$$

$$\Rightarrow \log y = \lim_{x \to 0} \frac{\log \sin x - \log x}{x^2} \quad \left[\frac{0}{0}\right]$$

Applying L-Hospital Rule

$$= \lim_{x \to 0} \frac{\frac{1}{\sin x}\cos x - \frac{1}{x}}{2x}$$

$$= \frac{1}{2} \lim_{x \to 0} \frac{x\cos x - \sin x}{x^2 \sin x} \quad \left[\frac{0}{0}\right]$$

Again Applying L-Hospital Rule

$$= \frac{1}{2} \lim_{x \to 0} \frac{\cos x - x\sin x - \cos x}{2x\sin x + x^2 \cos x} \quad \left[\frac{0}{0}\right]$$

$$= -\frac{1}{2} \lim_{x \to 0} \frac{\sin x}{2\sin x + x\cos x} \quad [0/0]$$

$$= -\frac{1}{2} \lim_{x \to 0} \frac{\cos x}{2\cos x + \cos x - x\sin x}$$

$$= -\frac{1}{2} \cdot \frac{1}{3} = -1/6$$

$$\log y = -\frac{1}{6}$$

$$\therefore y = e^{-1/6}$$